School on black holes and conformal boundaries of spacetime

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1 General Information

The school is devoted to the mathematical aspects of two dynamically developing and important areas of the general theory of relativity: conformal infinities of space-time and black holes.

Lecturers:

- Maciej Dunajski, University of Cambridge
- Dejan Gajic, University of Leipzig
- Wojciech Kamiński, University of Warsaw
- Jerzy Lewandowski, University of Warsaw
- James Lucietti, University of Edinburgh

Courses and lectures:

- Conformal methods in general relativity (Dunajski)
- Conformal geometry and the Fefferman-Graham construction (Kaminski)
- Normal Conformal Cartan Connection and asymptically (A)dS spacetimes (Lewandowski)
- Waves on black hole spacetimes (Gajic)
- Extreme black holes (Lucietti)
- Non-trivial NUT horizons (Lewandowski)

2 Detailed Program

Conformal methods in general relativity (Dunajski)

Abstract: The ten components of the Lorentzian metric in a four—dimensional space-time can be split into an overall scale function, and nine other functions which are scale invariant. Is it therefore the case that about 9/10th of general relativity and the associated mathematics does not care about the conformal factor? The aim of this mini—course is to make this question precise and provide some limited answers. It will start with the conformal compactifications of the Minkowski and Schwarzschild space—times, and proceed discuss three conformal invariant concepts:

- Causality of Scri in the presence of positive mass (aka the Penrose property),
- Conformal isometric embeddings of black holes,
- Conformal geodesics.

The Fefferman-Graham construction (Kamiński)

Abstract: The subject of this course will be the Fefferman-Graham ambient metric construction and its two applications in general relativity. The first application is a direct one: an expansion of the metric around the conformal boundary in the case of non-zero cosmological constant. The second involves a generalization (due to Michael Anderson) of an elegant method of Helmut Friedrich to show stability of de Sitter-like spaces. Preliminary plan of the course:

- Ambient metric expansion, Poincare-Einstein metrics (cases of even and odd dimensions).
- Holographic stress energy tensor, the obstruction tensor etc.
- Covariantly constant tractors and Einstein's equations. Geometric classification of solutions on conformally flat space-times.
- Harmonic extensions and classification of conformal invariants.
- Friedrich's method of showing stability of de Sitter space-times.
- Application of the obstruction tensor in showing stability of de Sitter space-time in higher even dimensions.

Conformal Cartan Connection and asymptically (A)dS space-times (Lewandowski)

Abstract: CCC is used to introduce a conformally invariant definition of symplectic gravitational field potential. It is regular on the conformal boundary of space-time, which is asymptotically (A)dS. This approach leads to a conformally invariant theory of gravitational field charges.

Waves on black hole spacetimes (Gajic)

Abstract: In recent years, there has been great progress towards a mathematical understanding of global properties of dynamical black holes arising from small perturbations of stationary black hole initial data, in the context of the Einstein equations. These developments are built upon a quantitative and robust mathematical understanding of geometric wave equations on fixed stationary black hole backgrounds. In these lectures, we will discuss the geometric wave equation on Schwarzschild and Kerr spacetimes and we will highlight the role of extremality of the black hole horizon in the dynamics.

Extreme black holes (Lucietti)

Abstract: The celebrated no-hair theorem in general relativity roughly states that an asymptotically flat, stationary, electro-vacuum black hole spacetime must be a Kerr-Newman solution. This remarkable uniqueness theorem was originally established for non-extreme black holes, although in recent years this has been extended to extreme black holes after an improved understanding of their near-horizon geometry. The aim of these lectures is to discuss some of these developments together with the necessary background material. Topics to be discussed:

- Black holes in General Relativity
- Extreme horizons and near-horizon geometry
- Rigidity of extreme horizons
- Uniqueness theorems for extreme black holes

Non-trivial NUT extreme and non-extreme horizons (Lewandowski)

Abstract: Kerr-NUT-(A)dS spacetimes and their accelerated generalization can be given the global structure of the Cartesian product of the Hopf bundle and real line. The doubly non-trivial structure of the Killing horizons contained therein will be discussed.